Force Balanced DC Transmission Lines

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Eric Dollard's DC Transmission Line Exercise

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Eric posted a transmission line puzzle. Here is my answer

************* Original Posting ***********************

I have a D.C. transmission line, the conductors are 2 inches in
diameter, spacing is 18 feet.

How many ounces of force are developed upon a 600 foot span of
this line, for the following;

1. 1000 ampere line current,

2. 1000 KV line potential?
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My Answer

First, find the magnetic repulsion between the two conductors by calculating B, then getting forces by $J \times B$.

Calculate B using the infinite line approximation.

$$
\begin{aligned}
B &= \mu_0 H = \mu_0 \frac{I}{2\pi r} \\
r &= 18 \text{ feet} = 5.48 \text{ m} \\
I &= 1000 \text{ A} \\
B &= (4\pi 10^{-7} * 1000)/(2\pi 5.48) = 11.6\ \mu\text{T}
\end{aligned}
$$

This field is small compared to terrestrial fields, which are around 100 μT.

We now calculate the force per length.

$$
\begin{aligned}
f &= \vec{j} \times \vec{B} \quad \text{body force} \\
F &= \int \vec{j} \times \vec{B} dA dl \\
&= I * B * l \\
&= 1000 \text{ A} \times 11.6\ \mu\text{T} \times 182.88 \text{ meter} \\
&= 2.12 \text{ N} \\
&= 7.62 \text{ oz}
\end{aligned}
$$

I have used 600' = 182.88m, and 1N = 3.586 oz. This force is small compared to gravity loads and windage loads.

Now we find the electrostatic attraction terms.

I use the principle of virtual work with parallel plate capacitors approximated by the 2 in diameter conductors separated by 18 feet. I model the capacitor as a flat ribbon with 18 feet separation. The curvature of the cylindrical conductor introduces a small error of the order 2in/18ft = 0.9%, so no problem.

$$
\begin{aligned}
E &= \frac{1}{2} C V^2 \\
&= \frac{1}{2} \frac{\epsilon A}{d} V^2
\end{aligned}
$$

where A = 2 inch by 600 ft, and d is 18 ft. Using the principle of virtual work, the attractive force between the capacitor plates is

$$\begin{aligned}
\vec{F} &= \nabla E \\
&= \frac{\partial}{\partial d}\left(\frac{1}{2}\frac{\epsilon A}{d}V^2\right) \\
&= -\frac{1}{2}\frac{\epsilon A}{d^2}V^2 \\
&= -(1/2)((8.854 * 10^{-12} * 182.88m * 0.0508m)/(5.48m * 5.48m))(10^6 V)^2 \\
&= -1.36955 \text{ N} = -4.92 \text{ oz}
\end{aligned}$$

Again, for this example, the electrostatic forces are neglible compared to gravity and windage.

Balance Magnitudes of Attraction and Repulsion

We can imagine a more compact system, where the electrostatic and magnetic forces become significant. We can balance mechanical forces from repulsion and attraction by operating at a specific current to voltage ratio. In effect, there will be a characteristic impedance associated with this force balanced system. We will have a transmission line in the classical sense.

$$\begin{aligned}
F'_{\text{mag}} &= F'_{\text{electrostatic}} \\
\frac{\mu_0 I^2}{2\pi}\frac{L}{d} &= \frac{1}{2}\frac{\epsilon(L * \text{WireDiameter})}{d^2}V^2 \\
\left(\frac{V}{I}\right)^2 &= \left(\frac{\mu}{\epsilon}\right)\frac{d}{\pi * \text{WireDiameter}} \\
Z &= 377\sqrt{\frac{d}{\pi * \text{WireDiameter}}}
\end{aligned}$$

If we operate our generator with the impedance prescribed voltage/current ratio, and terminate our load with the same impedance, we can have the force balanced transmission line.

Knowing Murphy's law, a system which needs to be critically balanced to operate (I'm thinking of a pulsed power system), would likely find a way to fail catastrophically.